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Dual-Mode Ultrasonic Apparatus for Measuring Compressional and Shear Wave Velocities of Rock Samples

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Abstract—**An ultrasonic apparatus and accurate experimental techniques have been developed for measuring consecutively the bulk velocity of plane compressional and shear waves in samples of porous solid material. The apparatus was developed primarily to study the elastic behavior of rock samples recovered from well bores in the earth as deep as 7800 meters. Appropriate conditions of triaxial stress in the range 0–25 000 lbf/in^2 (172 $\times$ 10^6 N/m^2), fluid saturation, and temperature are imposed on the sample to simulate the surroundings of the parent rock *in situ*.**

In the apparatus, two electromechanical transducer cells operating in the frequency range 0.25–5 MHz are placed in direct contact with each end of the sample. Phase-coherent tone-burst pulses are used. The cells contain two sets of piezoelectric ceramic disks. One set generates and detects compressional waves, and the other set serves as the source and detector for shear waves, which form by mode conversion at an oil–aluminum interface. The mechanical design of the apparatus is based upon sound principles of geometrical acoustics. This method has substantial advantages over existing ones for testing rocks because it has greater accuracy and a shorter testing time. Transit times through nondispersive rocks can be measured with a resolution of 10^{-8} second. The time formerly required for making separate measurements of compressional and shear wave velocities has been reduced by one half. Velocities and elastic moduli are reported for aluminum, Solenhofen limestone, and Boise sandstone samples under stress.

GLOSSARY OF SYMBOLS

P wave = compressional plane wave
S wave = shear plane wave
SV wave = component of S wave in the plane of propagation
V_p = velocity of P wave in solid
V_s = velocity of S wave in solid
V_1 = wave velocity in liquid
K = bulk modulus
$m_1 = m_2 = V_p/V_s$
$m_3 = K/(\rho_2 V_s^2)$
$n_1 = V_1/V_p$
$n_2 = V_p/V_1$
ρ_1 = density of liquid
ρ_2 = density of solid
$r_1 = \rho_1/\rho_2$
$r_2 = \rho_2/\rho_1$
$\bar{d}p$ = average particle diameter
ΔT_p = transit time of P wave
ΔT_{p1} = transit time of P wave through transducer cells (no sample)
ΔT_{p2} = transit time of P wave through transducer cells and sample
ΔT_s = transit time of S wave
ΔT_{s1} = transit time of S wave through transducer cells (no sample)
ΔT_{s2} = transit time of S wave through transducer cells and sample
Z_1 = acoustic impedance of transducer wedge material = $\rho_1 V_p$ for P-wave impedance or = $\rho_1 V_s$ for S-wave impedance
Z_2 = acoustic impedance of rock = $\rho_2 V_p$ for P-wave impedance or = $\rho_2 V_s$ for S-wave impedance
α = angle of incidence of P wave in liquid
β = angle of refraction of S wave in solid (in Fig. 1, β is also equal to angle of reflection of P wave in aluminum)
ϵ = angle of refraction of P wave in solid
ζ = angle of reflection of S wave in solid
η = angle of refraction of P wave in liquid
θ = angle of incidence of P wave in solid
λ = wavelength of carrier frequency of pulse

INTRODUCTION

ULTRASONIC pulse-propagation methods have been used extensively for measuring plane-wave velocities in the laboratory to study the elastic behavior of geological materials subjected to various environmental conditions. An understanding of the interrelationships between rock properties and environmental factors has a direct bearing on the success of geophysical prospecting methods in finding and delineating subsurface geological structures. The relationships are equally important in drilling, rock mechanics, and formation evaluation studies.

The rocks that are of most interest to the petroleum industry are potentially oil-bearing sedimentary types found as deep as 7800 meters. Usually, these rocks are granular and porous with interconnected fluid-filled interstices. The chief factors influencing the acoustic velocity of subsurface rocks are porosity, mineral composition, intergranular elastic behavior, and properties of the formation fluid. These factors depend upon overburden pressure, formation fluid pressure, temperature,

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microcracks, age, and depth of burial. In the laboratory, subsurface environments can be simulated realistically on samples taken from cores recovered from wells; appropriate conditions of stress, fluid saturation, and temperature are imposed on the samples. Wyllie *et al.* [1] found that compressional wave velocities measured on sandstone and carbonate samples under appropriate environmental conditions in the laboratory agree well with velocities measured on the parent rock by sonic well-bore-logging techniques. Many of the factors that influence the velocity of rocks have been studied experimentally. Results reported by Hughes *et al.* [2], Wyllie *et al.* [1], [3], Birch [4], [5], King [6], Gregory [7], and Podio *et al.* [8] are typical of the work done in this field.

The measurement of both compressional (P wave) and shear (S wave) velocities permits the dynamic elastic behavior of rocks to be studied in detail. However, a problem arises when separate laboratory experimental setups are required for measuring P wave and S wave velocities as a function of pressure. The first application of pressure may cause irreversible physical changes in the specimen. Stress history and hysteresis effects are known to play an important role in the elasticity of rocks. Gardner *et al.* [9] showed that the elastic response of a sedimentary rock to an imposed stress cycle depends somewhat on the previous stress cycles to which the rock was subjected. More realistic studies of elastic behavior can be made when P-wave and S-wave velocities are measured simultaneously or consecutively during the same stress cycle.

Other investigators have employed different techniques for simultaneous or consecutive determinations of P-wave and S-wave velocities in rock samples. Three methods have been used. The first method, described by Mason and McSkimin [10], makes use of longitudinal mode excitation of a transducer mounted on the end of a rod. This is the only method where both P waves and S waves are intentionally displayed simultaneously in the same wave train. A transducer on the opposite end of the rod detects the P-wave pulses that traverse the rod lengthwise. Also detected are S-wave pulses, which form by mode conversion at the wall and are delayed by one or more reflections across the diameter of the rod. The method was used extensively by Hughes and his coworkers [11]–[14], but Gregory [15] found that in rocks the onset of the shear wave trains was frequently ambiguous. In the second method, velocities are derived from critical angle measurements. Interesting results were reported by Subbarao and Ramachandra Rao [16], Wyllie *et al.* [17], King and Fatt [18], and Gregory [19]. These studies showed that the critical angle method has inherent limitations in accuracy and that interpreting results is too complex for routine laboratory tests. The third method relies on apparatus with P-wave and S-wave transducers mounted ingeniously where interference between modes is minimized or does not occur. Steveninck [20] and King [21] used a sandwich of two transducers. One generated S waves, the other one P waves. Desai [22] described an arrangement for mounting a P-wave ceramic tube on the same vertical axis above a small ac-cut quartz disk so that serious interference between transducers was avoided. Despite the unusual and interesting design features of the transducer mountings, the techniques used for measuring transit times are subject to errors that are not acceptable by present-day standards.

The purpose of this paper is to describe a new dual-mode ultrasonic apparatus and measurement technique, which permits quick determinations of transit times of P waves and S waves to a resolution of 10^{-8} second. This accuracy is necessary for evaluating the influence of factors that cause small variations in the wave velocity of rocks. Experimental results are presented for aluminum, Solenhofen limestone, and Boise sandstone. The apparatus and experimental techniques are readily adapted to an automated pulse measurement system with digital data outputs in a form suitable for computer analysis. Advantages of an automated system are discussed by Thill and Bur [23].

APPARATUS

General Principles

The dual-mode ultrasonic apparatus generates both P waves and S waves and directs them alternately through a sample in a single mechanical assemblage. The transducer assembly consists of two modified right-angle aluminum wedges placed in contact with the ends of a cylindrical solid sample as shown in Fig. 1. Wedge angles of 45° gave the most convenient geometry for guiding both P waves and S waves through the sample in the desired manner. To avoid wall reflections, the beams of energy were carefully directed along the vertical axis and parallel to the sides of the sample.

P waves originate at transducer S_1, which lies in direct contact with the flat aluminum wedge surface BF. Line NN' is drawn normal to the aluminum–oil interface BD. The beam travels at angle of incidence θ and is reflected from the interface along CR' at angle β where $\theta = 45° = \beta$. The P-wave energy continues through the sample and is detected by transducer S_1' after being reflected at point C' from interface $B'D'$. Equation (3) in the Appendix is useful for computing the amplitude ratios as a function of the angle of incidence for P waves reflected into aluminum at the aluminum–oil interface. Fig. 2 shows that the amplitude ratio is 0.46 when $\theta = 45°$. When the sample is removed and the wedges are placed face to face, the signal arriving at S_1' has an amplitude ratio of 0.21 after being reflected at two interfaces.

The well-known principles of mode conversion were used to obtain S waves in the dual-mode apparatus. A P-wave beam, incident in a liquid at a liquid–solid interface, is divided into P-wave and S-wave components, which are refracted at different angles in the solid according to Snell's law [(1) and (2) in the Appendix]. The relationships between the angles of incidence and the angles of refraction at an oil–aluminum interface are shown in Fig. 3. Most of the energy in the solid